Enrollment No: ______________________ **Exam Seat No:**______________________

C. U. SHAH UNIVERSITY

Winter Examination-2019

Subject Name : Discrete Mathematics

Subject Code : 4TE04DSM1 **Branch: B Tech (CE)**

Semester : 4 **Date : 01/10/2019** **Time : 02:30 To 05:30** **Marks : 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) The negation of "some students like football" is

(A) Some students dislike football (B) Every student dislikes football

(C) Every student likes football (D) none of these

b) If $\mathrm{T}(x): x$ is teach, $\mathrm{M}(x): x$ is male, then the symbolic representation of the statement "All teachers are male" is

(A) $\forall x\ (\mathrm{T}(x) \to \mathrm{M}(x))$ (B) $\forall x\ (\mathrm{T}(x) \vee \mathrm{M}(x))$

(C) $\forall x\ (\mathrm{T}(x) \wedge \mathrm{M}(x))$ (D) $\exists x\ (\mathrm{T}(x) \to \mathrm{M}(x))$

c) A binary operation on a set A is a mapping whose domain is

(A) $A \times A$ (B) A (C) set of integers (D) set of real numbers

d) Let G be a group and $a \in G$. If $\mathrm{O}(a) = 17$ then $\mathrm{O}(a^8)$ is

(A) 17 (B) 16 (C) 8 (D) 5

e) A group G is commutative iff

(A) $ab = ba$ (B) $(ab)^{-1} = b^{-1}a^{-1}$ (C) $(ab)^{-1} = a^{-1}b^{-1}$ (D) $(ab)^2 = ab$

f) Which of the following are posets?

(i) $(Z, =)$ (ii) $(Z, \neq)$ (iii) $(Z, >)$ (iv) $(Z, \geq)$

(A) (i) and (iv) (B) (i) and (ii) (C) (ii) and (iv) (D) (iii) and (iv)

g) A self-complemented, distributive lattice is called

(A) Boolean algebra (B) Modular lattice (C) Bounded lattice

(D) Complete lattice

h) In the lattice $\{1, 5, 25, 125\}$ with respect to the order relation divisibility, the complement of 1 is

(A) 1 (B) 5 (C) 25 (D) 125

i) If B is a Boolean Algebra, then which of the following is true

(A) B is a finite but not complemented lattice.

(B) B is a finite, complemented and distributive lattice.

(C) B is a finite, distributive but not complemented lattice.

(D) B is not distributive lattice.

j) The Boolean expression $A + AB + A\bar{B}$ is independent to:
(A) A (B) B (C) Both (A) and (B) (D) None of these

k) Another name for directed graph is ....
(A) Direct graph (B) Diggraph (C) Dir-graph (D) Digraph

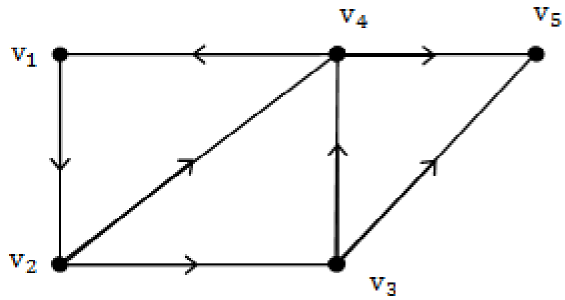
l) A graph is tree if and only if
(A) Is planar (B) Contains a circuit (C) Is minimally
(D) Is completely connected

m) Pigeonhole principle states that $A \to B$ and $|A| > |B|$ then:
(A) f is not onto (B) f is not one-one (C) f is neither one-one nor onto
(D) f may be one-one

n) Fuzzy logic is a form of
(A) Two-valued logic (B) Crisp set logic (C) Many-valued logic
(D) Binary set logic

Attempt any four questions from Q-2 to Q-8

Q-2		**Attempt all questions**	**(14)**
	a)	Show that □ r is a valid conclusion from the premises p $\Rightarrow$ □ q, r $\Rightarrow$ p, q (a) with truth table (b) without truth table.	**(5)**
	b)	Show that a subgroup H of a group G is normal if and only if $xHx^{-1} = H;\ \forall x \in G$.	**(5)**
	c)	Draw Hasse diagram for the poset $\langle S_{18},\ \mathbf{D} \rangle$; where $a\mathbf{D}b$ means a divides b.	**(4)**
Q-3		**Attempt all questions**	**(14)**
	a)	State and prove Lagrange's theorem on group.	**(5)**
	b)	Prove that $\langle \{1,\ 2,\ 3,\ 6\},\ \text{GCD, LCM} \rangle$ is a sublattice of the lattice $\langle S_{30},\ \text{GCD, LCM} \rangle$.	**(5)**
	c)	Find Meet-irreducible elements and antiatoms for the lattices $\langle S_{60},\ \text{D} \rangle$.	**(4)**
Q-4		**Attempt all questions**	**(14)**
	a)	Using definition of complement of an element find complement of each element of lattice $\langle S_{10},\ \text{GCD, LCM, 1, 10} \rangle$	**(5)**
	b)	Find all sub algebra of Boolean algebra $\langle S_{210},\ *,\ \oplus,\ ',\ 0,\ 1 \rangle$.	**(5)**
	c)	Draw all non-isomorphic graph on 2 and 3 vertices.	**(4)**
Q-5		**Attempt all questions**	**(14)**
	a)	State and prove Stone's representation theorem.	**(5)**
	b)	Draw the graph of tree represented by $\left(v_0\ \left(v_1\ (v_2)\ \left(v_3\ (v_4)\ (v_5)\right)\right)\ \left(v_6\ \left(v_7\,(v_8)\right)\ (v_9)\ (v_{10})\right)\right)$	**(5)**
	c)	Show that $3 + 33 + 333 + ... + 33...........3 = \left(10^{n+1} - 9n - 10\right)/27$ By mathematical induction.	**(4)**
Q-6		**Attempt all questions**	**(14)**
	a)	Find the node base of following of digraph.	**(5)**



b) Show that in any room of people who have been doing handshaking there will always be at least two people who have shaken hands the same number of times. **(5)**

c) Show that the following Boolean expression are equivalent. **(4)**

(i) $(x \oplus y) * (x' \oplus y),\ y$

(ii) $x * \left(y \oplus \left(y' * (y \oplus y')\right)\right),\ x$

(iii) $(z' \oplus x) * \left((x * y) \oplus z\right) * (z' \oplus y),\ x * y$

Q-7 **Attempt all questions** **(14)**

a) Let $a, b, c \in$ L and $\langle L, \leq \rangle$ be a lattice. Then prove that **(5)**

(i) $a \leq b,\ a \leq c \Rightarrow a \leq b * c,\ a \leq b \oplus c$

(ii) $b \leq a,\ c \leq a \Rightarrow b * c \leq a,\ b \oplus c \leq a$

b) Draw the graph where $\mathrm{V} = \{1, 2, 3, 4\}$ and $\mathrm{E} = \{e_1, e_2, e_3, e_4, e_5\}$, **(5)**

$e_1 = e_5 = (1, 2)$, $e_2 = (4, 3)$, $e_4 = (2, 4)$ and $e_3 = (1, 3)$.

c) Prove that $(Z_6,\ +_6)$ is a finite abelian group of order 6. **(4)**

Q-8 **Attempt all questions** **(14)**

a) Prove necessary and sufficient condition for a non-empty subset *H* of a group *G* to be a subgroup is that $a \in H,\ b \in H \Rightarrow ab^{-1} \in H$ where b^{-1} is the inverse of *b* in *G*. **(5)**

b) Find all the maxterms of a Boolean algebra with three variables x_1, x_2, x_3. **(5)**

c) Obtain the equivalent disjunctive normal form for the formula: **(4)**

$\square\ G \wedge (H \Leftrightarrow G)$